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Limit slenderness ratio of seismically isolated architectural heritage structures

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Summary

The article illustrates the determination of the maximum slenderness (the width to height ratio) of seismically isolated rigid structures of architectural heritage isolated by elastomeric bearings, depending on the stiffness of the bearings, horizontal design ground acceleration and the desired seismic force reduction of the fixed-base structure. The requirement that there should be no tensile stresses in the bearings was taken into account as a condition for determining maximum slenderness. A simple procedure has also been developed for the selection of bearings according to the above criterion which is suitable above all for the needs of elaborating preliminary design solutions, while for the elaboration of a final project it is necessary to take into account numerous other requirements from regulations and technical standards that ensure that the isolation will function correctly and protect the structure from damage. The use of this procedure is illustrated by the example of the Prešeren statue in Ljubljana.

1. Introduction and presentation of the research problem

The rehabilitation of architectural heritage is usually very demanding because of its high cultural value. Interventions affecting heritage buildings must be minimal or visible as little as possible, with the smallest possible influence on the aesthetics and functionality of the structure (ICOMOS, 1993 and 1999; Zupančič, 2007). According to the provisions of the Burra Charter (ICOMOS, 1999), new materials may be used in reconstructions, but it is desirable that as far as possible materials that are identical to the existing materials should be used. Such materials, however, frequently fail to meet the proper mechanical characteristics like more modern materials do, and thus do not make a significant contribution to increasing the earthquake safety of the reconstructed building. Frequently even the conceptual design of older buildings is unfavourable from the point of view of earthquake safety. From this point of view, seismic isolation represents a unique solution since it enables greater earthquake safety for heritage buildings with minimal intervention. The main function of seismic isolation devices is to increase the period of vibration of the

structure or reduce the level of forces that form in the structure during seismic loading (Skinner, 1993; Naeim and Kelly, 1999; Fischinger and Isaković, 2001; Christopoulos and Filiatrault, 2006).

For the seismic isolation of existing buildings, elastomeric isolators (rubber bearings) that can be combined with various dampers are often used (Eggert, 2002; Martelli, 2004). Designing the seismic isolation of a building is relatively demanding since this is a dynamic system in which the vibration of both the superstructure and the base-isolation system plays an important role (Martelli, 2004). An unsuitably designed system can do more harm than good if the period of vibration of the structure coincides with the prevailing periods of the earthquake and thus produces a resonant response (Kilar and Koren, 2007a). Seismic isolation has the greatest effect in structures whose basic period of vibration is in the plateau of the response spectrum (constant acceleration branch). In this case, by extending the period of vibration we reduce seismic forces several times. By contrast, it is necessary to be considerably more careful with structures that have very small periods of vibrations (smaller than $T_c = 0.1$ or 0.2 seconds – depending on the ground type), since through seismic isolation we can move them into an unfavourable resonance area.

The general requirements that must be met when designing bearings are set out in the European building code Eurocode 8 (CEN, 2004) and are also described and explained in detail in other technical literature: Skinner et al. (1993), Naeim and Kelly (1999) or, for example, Komodromos (2000). The general requirements are not the subject of this paper; this paper only deals with the determination of the maximum slenderness of the structure that can be placed on isolators so that no disconnection from the bearings or, consequently, overturning will occur. In the case of heavier and low-rise buildings, this slenderness will not, of course, be decisive, but it can have a significant influence on the choice of bearings in lighter, narrower and taller buildings, where the danger of uplift from the bearings is considerably greater.

In buildings with a greater ratio between height and width, seismic forces can cause rocking, which can lead to uplift from the bearings or the overturning of the seismically isolated structure (Li and Wu, 2006; Hino et al., 2008). Tensile stresses cause small cracks in the bearings that grow slowly under cyclical loading and significantly reduce the vertical and horizontal stiffness of the bearings (Skinner et al., 1993). Many authors, for example Skinner et al. (1993), Kelly (1997), and Li and Wu (2006), therefore recommend that the tensile strength of bearings should be neglected at the design stage, meaning that bearings should be designed as though they are not capable of supporting tensile loads.

2. Method of determining limit slenderness

In Figure 1 we show some typical heritage structures with different ratios of height (H) to width (B), which represents the shorter ground plan dimension of the structure. Because of the different geometry and materials used, the position of the centre of gravity of structures can vary greatly. In order to determine slenderness we therefore consider

three different centres of gravity (positions of masses), at 1/3, 1/2 and 2/3 of the total height of the structure above the ground (Figure 2), where we assumed that the structures in question are symmetrical with regard to the vertical axis. In the distribution of the bearings we assumed that the bearings are symmetrically distributed with regard to the vertical centroid axis and, in terms of the ground plan, distributed in rows, forming an orthogonal grid (Kilar and Koren, 2007b; Contento and Egidio, 2008). We also assumed that the superstructure is a rigid body and that the effective period of vibration of the superstructure (T_{eff}) is equal to the period of vibration (T) of the whole system. We believe that the above assumption is reasonable for heritage buildings, which are usually quite rigid (solidly built of stone or brick), but in terms of elasticity cannot tolerate major loads and tensile stresses. We considered three ground types – good, medium and poor. The ground types correspond to ground types A, B and D in Eurocode 8 (CEN, 2004). In all cases we took into account the fact that a system of base-isolation usually has 10% damping ($\xi = 10\%$ of critical damping). Bearings made from the latest elastomeric materials can of course achieve greater levels of damping (Christopoulos and Filiatrault, 2006).

As a condition for determining the maximum slenderness (H/B), the requirement that tensile stresses should not occur in the bearings was taken into account (the resultant of vertical forces falls onto the edge of the cross-section core). In many bearings, this condition is relatively demanding because tensile stresses can occur quite quickly when the eccentricity of the resultant reaches a value of 1/6 of the width of the foundation, while overturning of the system occurs considerably later, when the eccentricity of the resultant reaches a value of 1/2 of the width of the foundation and it is no longer possible to establish equilibrium. Checking the occurrence of tensile stresses in the bearings at the edge therefore represents a condition that is as strict as or stricter than the overturning condition. The fundamental parameters affecting limit slenderness are: mass (m), the effective stiffness of the isolators in horizontal direction (K_{eff}) and the effective period of vibration of the system (T_{eff}). Their correlation is expressed by Equation 1. An important role is also played by the height of the centre of gravity of the structure and the response spectrum, which is dependent on the quality of the ground.

$$K_{eff} = 4\pi^2 \frac{m}{T_{eff}^2} \quad (1)$$

We determined limit slendernesses with the help of linear analysis with the Eurocode 8 response spectrum. We calculated the maximum slenderness ($\max H/B$) depending on the effective period of vibration of the structure and design ground acceleration (a_g). We took into account, in the calculation, the fact that only two bearings lie in a single line across the width (B) of the structure – one on the far left, and the other on the far right. The number of rows of bearings along width B is therefore equal to 2. In this case the criterion of prevention of tensile stresses in the bearings gives the same results as the criterion of overturning.

3. Results of the method

Selected results are shown in Figure 3. The figure shows the maximum permitted slendernesses (H/B) for a period of vibration for design ground acceleration ranging from $a_g = 0.1$ g to $a_g = 0.5$ g, with a step of 0.1 g. Results are shown for three selected mass heights and three selected ground qualities. The lower section of Figure 3 also shows response spectra of displacements for individual cases, from which we can read the maximum displacement of the ground for the selected period of vibration. It is clear from Figure 3 that the condition of formation of tensile stresses can be decisive for the choice of bearings in cases where the ground is poor, the centre of gravity of the structure is high above the ground, and design ground acceleration is high.

In the cases where we have more than two rows of bearings, the criterion of prevention of tensile stresses in bearings on one edge of the structure becomes decisive. We incorporated the effect of this criterion with a correction factor (k_v), assuming an orthogonal and equidistant distribution of bearings. The correction factor can be expressed as presented in Equation 2, where v is the number of rows of bearings along the width of the structure (B). The maximum slenderness (H/B) must be multiplied by this factor. The correction factor is limited downwards by 1. If we were to further increase the number of rows of bearings, we would see that this factor is also limited upwards, by a value of 3.

$$k_v = 3 - \frac{6}{v+1} \quad (2)$$

It must be stressed that in many cases the occurrence of tensile stresses in bearings is not a decisive condition for the selection of bearings. In other cases again, base-isolation does not make sense if it does not also produce a significant reduction of seismic forces in comparison with a fixed-base structure, since damage may still occur in the superstructure in the case of an earthquake. For the purpose of estimating the effectiveness of base-isolation, Figure 3 shows the reduction factors (RI) that present a reduction of seismic forces as a result of the influence of seismic isolation. Thus for example in the case of a reduction $RI = 2$, we get a halving of seismic forces. The use of bearings that give small reduction factors and large ground displacements does not of course make sense from the design point of view, nor is it justifiable in terms of cost.

4. Analysis and use of results

Below is a presentation of a simple procedure for estimating the necessary stiffness of bearings according to the criterion of slenderness:

1. For the structure in question we determine the slenderness (H/B).
2. We determine the mass of the whole system (m), which includes the mass of the structure and the mass of the isolation system, and the position of the mass (position of the centre of gravity).

3. With regard to the location of the structure, we determine the type of ground and the design ground acceleration (a_g).
4. We select the total number of bearings (n) and the number of rows of bearings (v).
5. From Equation 2 we calculate the correction factor (k_v) for the selected number of rows of bearings (v).
6. We determine the corrected slenderness (H/B) by multiplying the actual slenderness by the factor k_v .
7. From Figure 3, on the basis of the corrected slenderness (H/B), the selected ground type, the design ground acceleration and the position of the mass, we read the corresponding necessary effective period of vibration of the system (T_{eff}).
8. From Equation 1 we calculate the necessary effective stiffness of the isolation system (K_{eff}).
9. From the displacement spectrum (Figure 3) we read the displacement of the entire system.
10. We determine the necessary effective stiffness of one bearing ($K_{eff,i}$).
11. With regard to the necessary effective stiffness of the bearing and the displacement of the whole system, we select a suitable bearing.
12. We verify whether the selected bearing meets general requirements for designing bearings. The general requirements that must be met in designing bearings are set out in Eurocode 8 (CEN, 2004).

We can also proceed in the opposite direction by first selecting the stiffness of the isolation system and the number of rows of bearings. From Equation 2 we calculate the appropriate correction factor and then from Figure 3 we read off the maximum permitted corrected slenderness (H/B) for a given ground type and given design ground acceleration. We divide the corrected slenderness (H/B) by the correction factor in order to get the actual maximum slenderness (H/B). If the slenderness is not suitable, we select a different bearing with a smaller effective stiffness and repeat the procedure. This procedure is also useful if the maximum force that the superstructure can sustain is prescribed. In this case a RI factor is prescribed, from which we can calculate T_{eff} and only then verify that the danger of overturning is excluded.

5. Example of use

The Prešeren statue in Ljubljana is presented as an example of use (Figure 4). In accordance with the procedure described in the previous paragraph, we first determine the slenderness (H/B). The basis for the choice of slenderness is indicated in the light-coloured box in Figure 4. We assume that the steps around the base are not structurally connected to the statue. In this case the slenderness $H/B = 3$. We estimate the mass of the complete system at 30 tonnes and assume that the centre of gravity of the statue is situated at $1/3$ of its height. For the location we select a medium ground and a ground acceleration (a_g) of $0.5 g$. We select four bearings distributed around the ground plan in the form of a square, i.e. in two rows ($n = 4$, $v = 2$). From Equation 2 we calculate the correction factor, which for

two rows of bearings amounts to $k_v = 1$. From Figure 3 we then read the corresponding effective period of vibration of the system, which for the selected slenderness and ground type amounts to $T_{\text{eff}} = 1.15$ s. The reading process is also illustrated separately in Figure 5 (red arrows), which contains a graph for the maximum slenderness (H/B) in relation to the period of vibration and the spectrum of movements for the selected medium ground and a mass position at $1/3$ of the height of the structure.

We calculate the effective stiffness of the isolation system from Equation 1 and get the result $K_{\text{eff}} = 822$ kN/m. The effective stiffness of one bearing thus amounts to $K_{\text{eff},i} = 205$ kN/m. From the displacement spectrum (Figure 5), we read the displacement of the entire system, which amounts to 19 cm. Finally, we select bearing SI-S 300/100 from the catalogue. This bearing is capable of sustaining a maximum movement of 20 cm (FIP Industriale, 2008). In this case the reduction of seismic forces as a result of the functioning of seismic isolation amounts to approximately $RI = 2.4$. If we had chosen a longer isolation time, the reduction of forces would be greater, but so would the horizontal movement.

If instead of four bearings we choose six bearings in three rows ($n = 6$, $v = 3$), Equation 2 then gives us a correction factor (k_v) equal to 1.35. We must then multiply the actual slenderness (H/B) by 1.35. We get a corrected slenderness (H/B) that amounts to 4.05. For this slenderness we read off the corresponding effective period of vibration of the system, which amounts to $T_{\text{eff}} = 1.6$ s (Figure 5, green arrows). The total movement of the system amounts in this case to 24 cm, while the reduction of seismic forces is $RI = 3.2$. In accordance with the provisions of the European standard Eurocode 8, other additional controls and appropriate analysis still need to be carried out for the chosen bearing to confirm the presented choice. These must be carried out by a specialist planner.

Figure 5 also shows the influence of the elasticity of the superstructure on the effective period of vibration for various slendernesses (blue hatched line). An example is shown where the period of vibration of the superstructure is equal to 0.5 s (at the end of the plateau in the response spectrum). We can see that in this case for a slenderness of $H/B = 3$ we would have to choose an effective period of vibration $T_{\text{eff}} = 1.2$ s that is less than the period of vibration which we selected above. This means that the influence of the elasticity of the superstructure is small and that in all cases it enables the use of greater slenderness, and therefore has a favourable influence.

All the analyses covered were done using the response spectrum from Eurocode 8 (CEN, 2004), taking into account only the horizontal component of ground acceleration. It would actually also be necessary to take into account the vertical component of an earthquake, where the correlation of seismic waves is also important, since it is not necessarily the case that maximum accelerations occur simultaneously in the horizontal and vertical directions. We are not able to incorporate this influence accurately when using the response spectrum calculation. In such cases the Eurocode 8 demands a combination of seismic effects where, together with the horizontal response, only a 30% value of the vertical component of ground acceleration is taken into account. For such cases an additional curve of maximum slenderness (H/B) is shown in Figure 5. We can see that as a result of the operation of the

vertical component the maximum slendernesses are slightly lower and that the vertical component of the earthquake has an unfavourable effect. This combination is not always completely reliable (Hernández and López, 2003). In order to determine the more precise influence of the vertical component it would be necessary to carry out dynamic analysis taking into account the simultaneous effects of the horizontal and vertical components of the time-history records of accelerations for an individual earthquake (or group of earthquakes) with characteristics known in advance, something which was not done in this phase of the research.

6. Discussion and conclusions

For structures with different ratios between height, width and position of masses on ground types of different quality, we showed the maximum permitted slenderness of the structure or cultural monument that we can place on rubber bearings without tensile stresses forming in them as a result of the horizontal seismic loading. We determined limit slendernesses with the help of linear analysis with the Eurocode 8 response spectrum. We calculated the maximum slenderness ($\max H/B$), depending on the effective period of vibration of the structure and the design ground acceleration (a_g). We may conclude that the condition of the occurrence of tensile stresses is decisive for the choice of bearings in cases where the ground is poor, the centre of gravity of the structure is high above the ground, and design ground acceleration is high. In other cases the occurrence of tensile stresses in bearings is not a decisive condition for the choice of bearings.

We also see from the results of the analyses that the quality of the ground and the position of the mass have a significant influence on the maximum slenderness (H/B) with a specific period of vibration or with a specific reduction of seismic forces. With the same reduction we can have a 50% greater maximum slenderness (H/B) on a good ground than on a poor ground. We can also see that slenderer structures on seismic isolation are less exposed to overturning, since the isolation reduces horizontal forces in comparison to forces on a fixed-base structure. It is generally the case that isolation does not make sense if it does not also bring a significant reduction of seismic forces in comparison with a fixed-base structure, since damage may still occur to the superstructure in the case of an earthquake. The use of bearings that give small reduction factors does not of course make sense from the design point of view, nor is it justifiable in terms of cost. On the other hand bearings with which we achieve very high reduction factors also have relatively high period of vibrations and, consequently, large displacements. Relatively large displacements of seismically isolated structures can be problematic, and it is therefore also necessary to consider this problem in preliminary studies and on this basis choose a suitable reduction factor.

A simple procedure for the selection of the period of vibration of bearings (stiffness of bearings) is also illustrated. This procedure is useful for estimating the dimensions of bearings and evaluating the efficiency of isolation in an individual case. It is above all suitable for the needs of elaborating preliminary design solutions, while for the elaboration of final projects it is also necessary to consider numerous other requirements from regulations and

technical standards that ensure that the isolation will function correctly and protect the structure from damage.

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